Formulate the following problem as an LP model. DO NOT SOLVE. The Soundex Company produces two models of radios. Model A requires 15 minutes of work on assembly line I and 10 minutes ON work on assembly line II. Model B requires 11 minutes of work ON assembly line I and 12 minutes of work on assembly line II. At most 25 hours of assembly time is available on Line I and at most 22 hours on line II, per day. The company will realize a profit of \$ 6 per unit on model A and \$ 8 per unit on model B. How many radios of each type should be produced per day in orde to maximize the profit?

Soln

		A		B		
	Max P =	$6x_1$	+	$8x_2$		
I		$15x_1$	+	$11x_2$	$\leq$	1,500
II		$10x_1$	+	$12x_2$	$\leq$	1,320

x_1 = # MODEL A RADIOS TO MAKE PER DAY
x_2 = " " B " " " " "

$x_1, x_2 \geq 0$

SET-UP the following as an LP problem (DO NOT SOLVE).

A company makes rackets for tennis, squash, and racketball. Each tennis racket requires 2 units of aluminum and 1 unit of nylon. Each squash racket requires 1.8 units of aluminum and 0.8 units of nylon. Each racketball racket requires 1.5 units of aluminum and 1.1 units of nylon. The company has 1000 units of aluminum and 800 units of nylon available per week. The company is not able to manufacture more than 550 rackets in total per week, but must manufacture at least 50 squash rackets per week. Because of the popularity of tennis, each week the company must make more tennis rackets than squash rackets and racketball rackets combined. The profit on each tennis, squash, and racketball racket is $7, $9, and $10, respectively. How many rackets of each type should the company make in order to maximize weekly profit?

$x_1 > x_2 + x_3$

$$\max P = 7x_1 + 9x_2 + 10x_3$$

$$2x_1 + 1.8x_2 + 1.5x_3 \leq 1000$$

$$x_1 + 0.8x_2 + 1.1x_3 \leq 800$$

$$x_1 + x_2 + x_3 \leq 550$$

$x_1 > x_2 + x_3 \rightarrow$ $$x_1 - x_2 - x_3 > 0$$

$$x_2 \geq 50$$

$$x_1, \cancel{x_2}, x_3 \geq 0$$

x_1 = # TENNIS RACKETS TO BE MADE PER WEEK

x_2 = " SQUASH " " " " " "

x_3 = " RACKETBALL " " " " " "

SET UP, DON'T SOLVE

Mr. Jones wishes to consume at least 200 mg of vitamin C and 1000 units of Vitamin A each day on the theory that it will help reduce the threat of cancer. He decides to get these vitamins by eating strawberries and raspberries since he likes them and they are both good sources of the vitamins. One cup of strawberries has 90 units of vitamin A, 88 mg of vitamin C, and costs $0.85. One cup of raspberries has 160 units of vitamin A, 31 mg of vitamin C, and costs $1.35. If he wishes to meet his vitamin requirement at minimum cost, how many cups of each fruit should he eat each day?

SOLN

STRAWBERRIES vs. RASPBERRIES

$$\min C = .85x_1 + 1.35x_2$$

$$\text{if} \quad 90x_1 + 160x_2 \geq 1000$$

$$88x_1 + 31x_2 \geq 200$$

$$x_1 \geq 0,\ x_2 \geq 0$$

Let x_1 = the number of cups of strawberries he should eat per day

x_2 = the number of cups of raspberries he should eat per day

LO-FAT FOODS makes two types of artificial sweeteners, called SWEET and LO-SUGAR, by blending saccharin and dextrose. Each kilogram of SWEET needs 0.2 kg of saccharin and 0.4 kg of dextrose while each kilogram of LO-SUGAR needs 0.5 kg of saccharin and 0.3 kg of dextrose. The company has 25 million kg of saccharin and 31 million kg of dextrose on hand. The profit on one kg of SWEET is \$0.45 while the profit on one kg of LO-SUGAR is \$0.65. If the company can sell all that it makes, how much of each product should it make to maximize its profit?

SOLN

LO-FAT FOODS

	SWEET	LO-SUGAR	co-has
SAC	0.2	0.5	25 000000
DEX	0.4	0.3	31 000000
profit \$	0.45	0.60	

$\rightarrow$

$$\begin{cases} \max P = 0.45x_1 + 0.60x_2 \\ 0.2x_1 + 0.5x_2 \le 25{,}000{,}000 \\ 0.4x_1 + 0.3x_2 \le 31{,}000{,}000 \\ x_1, x_2 \geq 0 \\ x_1 = \text{\# kg OF SWEET to make} \\ x_2 = \text{\# kg OF LO-SUGAR to make} \end{cases}$$

An insurance firm wishes to invest \$250 million without taking too many risks. The managers estimate that they can earn interest of 11% in stocks, 9% in bonds, and 6.5% in treasury bills. To limit the risks, they decide to invest at least twice as much in treasury bills as in stocks and bonds combined and at least as much in bonds as in stocks. How should they distribute their investment to maximize their anticipated income for the year?

SOLN

$$\begin{array}{ll}
\text{max} & P = .11x_1 + .09x_2 + .065x_3 \\
\text{if} & x_1 + x_2 + x_3 \leq 250 \qquad (= 250 \text{ can be justified}) \\
 & -2x_1 - 2x_2 + x_3 \geq 0 \\
 & -x_1 + x_2 \geq 0 \\
 & x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 \geq 0 \\
\text{where} & x_1 = \text{ number of million dollars invested in stocks} \\
 & x_2 = \text{ number of million dollars invested in bonds} \\
 & x_3 = \text{ number of million dollars invested in treasury bills}
\end{array}$$

Sam is on a special diet and needs two food supplements (I and II) which he can get from two different products (A and B). Each serving of A has 3 units of I and 2 units of II. Each serving of B has 1 unit of I and 4 units of II. Sam needs 10 units of I and 12 units of II each day. If a serving of A costs \$0.50 and a serving of B costs \$0.40, how many servings of each product should he eat each day to meet his needs at minimum cost?

SOLN

$$\begin{array}{ll}
\text{min} & C = .50x_1 + .40x_2 \\
\text{if} & 3x_1 + x_2 \geq 10 \\
 & 2x_1 + 4x_2 \geq 12 \\
 & x_1 \geq 0, \quad x_2 \geq 0 \\
\text{where} & x_1 = \text{ number of servings of product A} \\
 & x_2 = \text{ number of servings of product B}
\end{array}$$

Formulate (set-up) the following as an LP problem (DO NOT SOLVE).

A company makes two sizes of wooden boxes, large boxes and small boxes. The large boxes require 10 minutes of labor and 12 square feet of wood, the small boxes 7 minutes of labor and 8 square feet of wood. The profit made on the larger ones is \$3 per box, on the smaller is \$1 per box. Each week there is 100,000 square feet of wood available. The company employs 5 workers who each work a 40 hour week. Each week the company supplies a nearby factory with 500 small boxes. How many boxes of each type should the company make in order to maximize weekly profit? (Hint: 40 hours=2,400 minutes.)

SOLN

	LARGE	SMALL	COMPANY
TIME (MIN)	10	7	12,000
WOOD (SQ. FT)	12	8	100,000
PROFIT (\$)	3	1	max

$$\max\ P = 3x_1 + x_2$$
$$10x_1 + 7x_2 \le 12{,}000$$
$$12x_1 + 8x_2 \le 100{,}000$$
$$x_1 \ge 0,\quad x_2 \ge 500$$
$$\cancel{(x_2 \ge 0)}$$

x_1 = # LARGE BOXES TO BE MADE / WEEK
x_2 = # SMALL " " "

Formulate each of the following problems as LP models. DO NOT SOLVE.

The Woodshop Company makes three products, which are called I, II, and III, and which earn a profit of \$23, \$29, and \$19 respectively. The products require 7, 5, and 2 board feet of lumber, respectively, and it takes 2.2, 3.4, and 1.1 hours of labor to construct each type, respectively. If there are 250 board feet of lumber and 95 hours of labor available on a particular day, how many of each type product should Woodshop make that day in order to maximize the profit?

max $\quad P = 23x_1 + 29x_2 + 19x_3$

if $\quad 7x_1 + 5x_2 + 2x_3 \le 250$

$\quad 2.2x_1 + 3.4x_2 + 1.1x_3 \le 95$

$\quad x_1 \ge 0, \quad x_2 \ge 0, \quad x_3 \ge 0$

where $\quad x_1 =$ number of product I made per day

$\quad x_2 =$ number of product II made per day

$\quad x_3 =$ number of product III made per day